Vector Calculus 20E, Spring 2012, Lecture B, Midterm 1

Fifty minutes, four problems. No calculators allowed.

Please start each problem on a new page.

You will get full credit only if you show all your work clearly.

Simplify answers if you can, but don't worry if you can't!

1. Calculate the integral of $f(x, y) = xe^{-y}$ over the triangle in $\mathbb{R}^2$ formed by the lines $x = 0$, $y = 0$, and $x + y = 1$.

2. Calculate the integral of the function $f(x, y, z) = z$ along the curve given by $(t, \frac{2}{3}t^{\frac{3}{2}}, t)$, where $2 \leq t \leq 7$.

3. Let $D^* = \{(u, v) : u^2 + v^2 \leq 1\}$ and let D be the image of D^* under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $x = u^3, y = v^3$. Calculate the area of D. (Hint: the following formulae may be useful.)

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

4. Use spherical polar coordinates to calculate the integral of the function $f(x, y, z) = z^2$ over the region of $\mathbb{R}^3$ between the spheres of radius 1 and 2.

Vector Calculus 20E, Spring 2013, Lecture A, Midterm 1

Fifty minutes, four problems. No calculators allowed.

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Simplify answers if you can, but don't worry if you can't!

1. Let $g : \mathbb{R}^3 \to \mathbb{R}^2$ be a map such that $g(0,0,0) = (2,3)$ and whose derivative Dg at $(0,0,0)$ is given by the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}.$$

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = xy - x + y$. Calculate the derivative $D(f \circ g)$ at $(0,0,0)$.

2. Let D be the region between the graphs of $y = x^2$ and $y = x^3$. Compute the integral

$$\iint_D xy \, \mathrm{d}A.$$

3. Let $D^* = \{(u,v) : 0 \le u \le 1, \ 0 \le v \le 1\}$ and let D be the image of D^* under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(u,v) = (u^2 v, uv^2)$. Calculate the area of D.

4. Let $f(x,y) = x \log y$. Find the approximation to the value of $f(1.03, 1.02)$ given by the second-order Taylor expansion of f at $(1,1)$.